Edexcel International AS/A Level & International GCSE

Teaching and Learning Strategies in Mathematics

Event Code: YMA01_20IF1

First teaching in 2018, first assessment 2019

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Session Agenda

10:00 Welcome & Introductions. Aims and Objectives
Pearson as a leading Awarding and training body.
10:10 How an examination assesses student performance
11: 25 Break & Networking activity
11:40 What students do in Edexcel maths examinations
12:10 What students need to do to raise achievment
1:00 Lunch
2:00 Improving student readiness and preparation for exams
2:30 How to do problem solving and carry out proofs
3:00 Organising teaching to raise achievement
3:50 Final questions

Aims and objectives

Delegates will:

- learn from analysis of how students have performed in examinations
- identify those areas of learning which students have found most challenging
- discuss the implications of that analysis for teaching and learning strategies
- be introduced to a range of teaching and learning strategies particularly applicable to mathematics
- discuss strategies for optimising the learning of students in mathematics
- network, discuss best practice and share ideas with other teachers.

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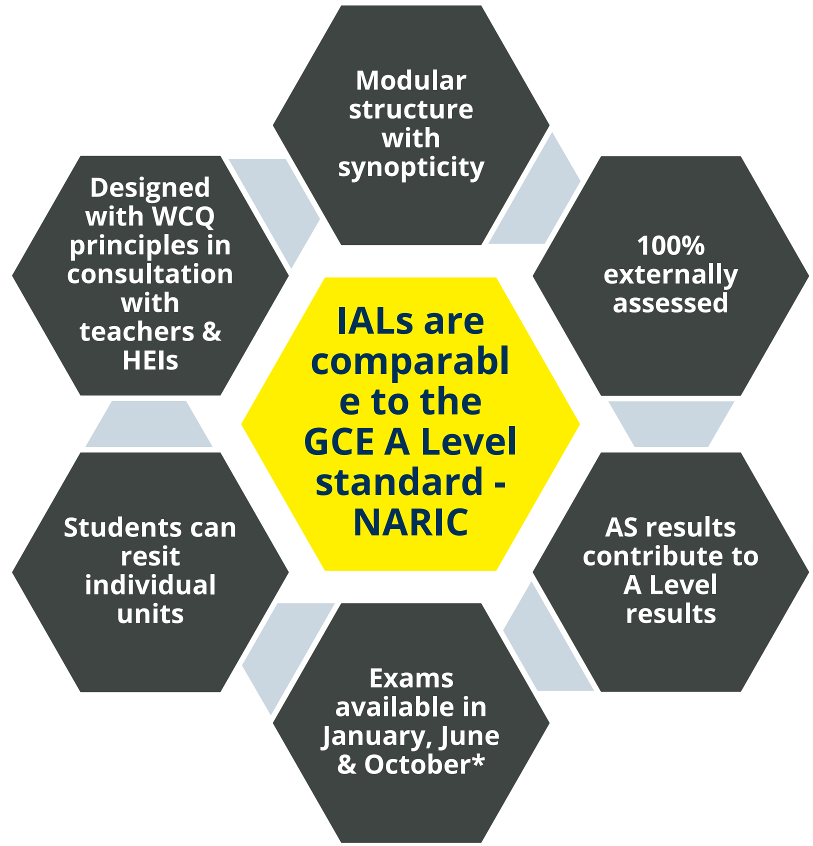
IAL Features

- International A Levels and AS Levels are created for International Students
- Globally recognised.



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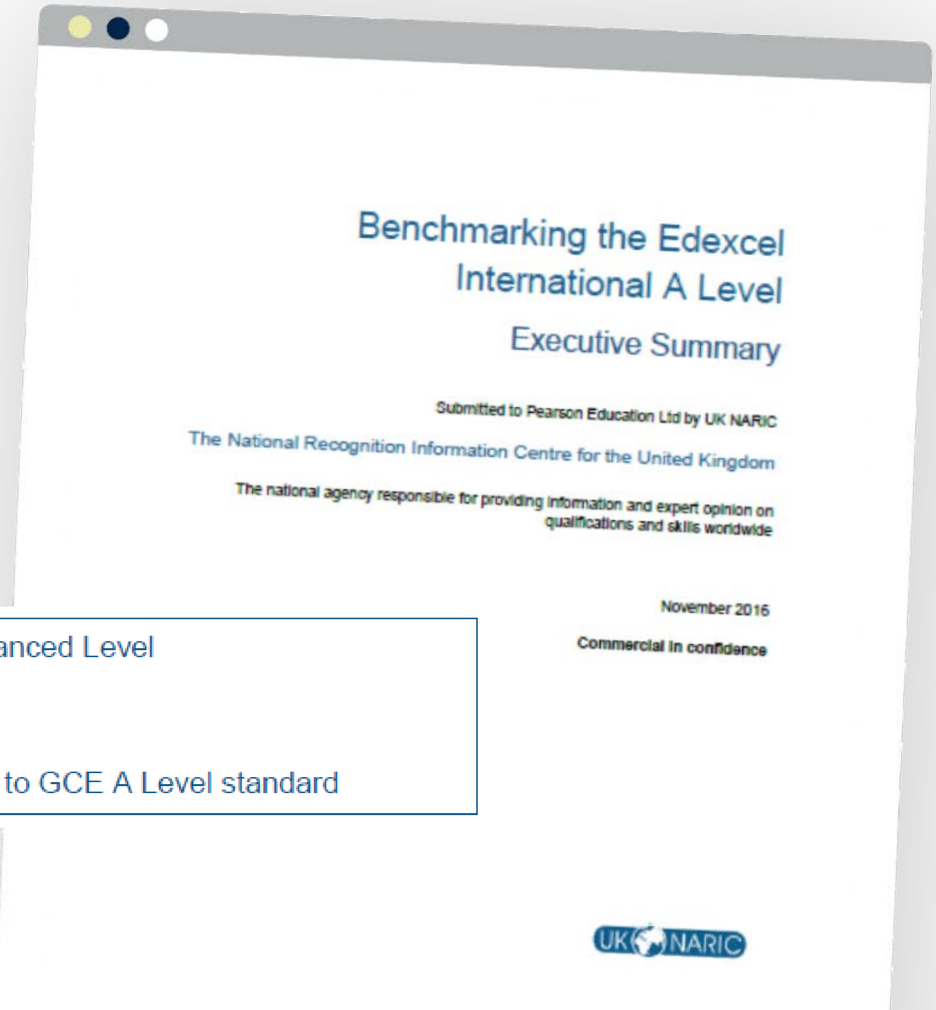
Updated NARIC report for Edexcel IAL

The executive summary confirms that Edexcel IALs are considered comparable to the GCE A Level standard following reforms to the UK regulated qualifications.


Benchmarking the Edexcel International A Level

Executive Summary

Submitted to Pearson Education Ltd by UK NARIC

The National Recognition Information Centre for the United Kingdom

The national agency responsible for providing information and expert opinion on qualifications and skills worldwide

November 2016

Commercial in confidence

UK NARIC

Qualification:	Edexcel International Advanced Level
Awarding Institution:	Pearson Education Ltd
Comparability:	Is considered comparable to GCE A Level standard

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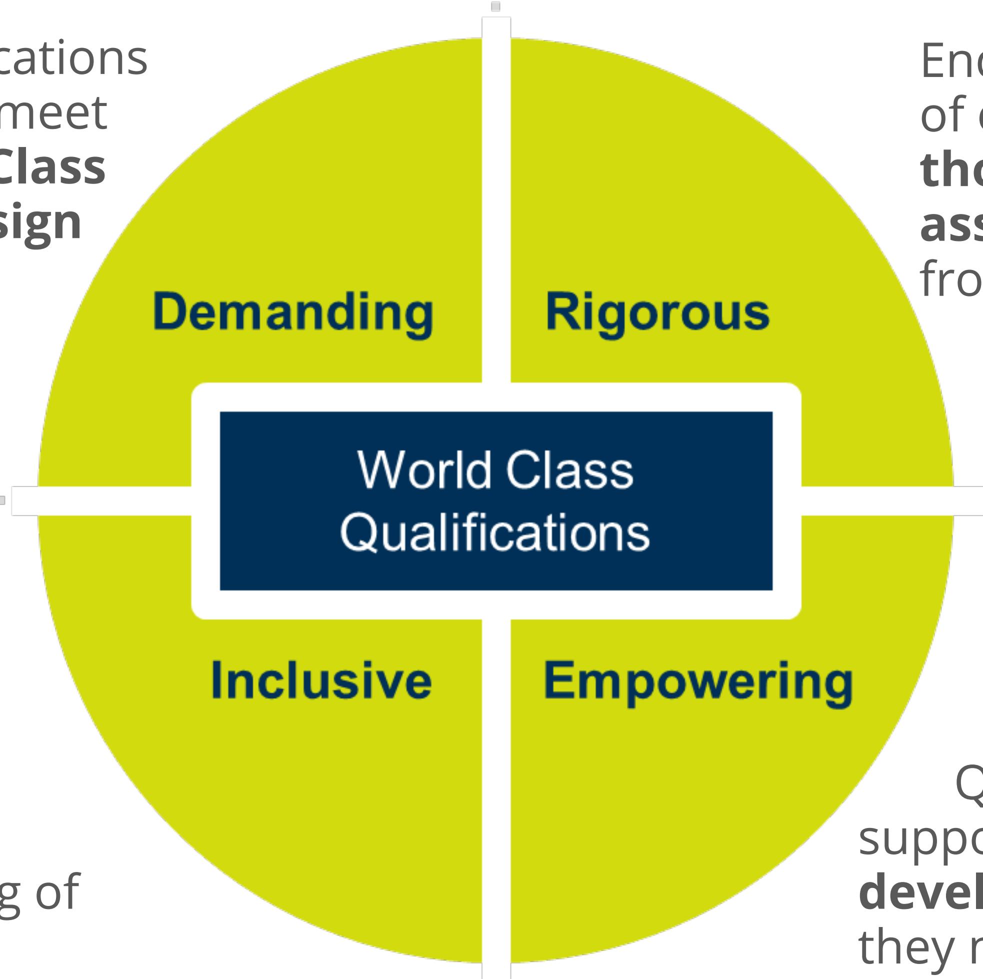
World-class qualifications

All Edexcel qualifications are developed to meet Pearson's **World Class Qualification design principles**

Endorsement of educational **thought-leaders and assessment experts** from across the globe



Developed using an understanding and benchmarking of **all educational systems**

Qualifications that support young people to **develop the capabilities** they need to **progress** and prosper in their lives

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How an examination assesses student performance

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Examinations

The defining features of an examination are:

- Content
- Assessment objectives
- Demand

Usually 'time' is not a major issue with our maths papers

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Examinations

Content:

- Given in the specification – facts and techniques
- These can be specific

P1.3 Unit content

What students need to learn:		Guidance
1. Algebra and functions		
1.1	Laws of indices for all rational exponents.	$a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$ The equivalence of $a^{\frac{m}{n}}$ and $\sqrt[n]{a^m}$ should be known.
1.2	Use and manipulation of surds.	Students should be able to rationalise denominators.
1.3	Quadratic functions and their graphs.	

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Examinations

Content:

- Given in the specification – facts and techniques
- Or more general ....

P2.3 Unit content

What students need to learn:		Guidance
1. Proof		
1.1	Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof stated below:	

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Examinations

Content then consists of:

- facts to be learned
- techniques to be learned
- concepts to be learned

Examinations

Assessment Objectives (AOs)

- The means by which the examination is written so that the student demonstrates their ability
- 5 AOs in A level mathematics and 3AOs in International GCSE
- The number of marks allocated to each AO is roughly the same for each examination series

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Examinations

- AO1 and AO2 have the most marks allocated at A level

		Minimum weighting in IAS	Minimum weighting in IA2	Minimum weighting in IAL
AO1	Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of contexts.	30%	30%	30%
AO2	Construct rigorous mathematical arguments and proofs through use of precise statements, logical deduction and inference and by the manipulation of mathematical expressions, including the construction of extended arguments for handling substantial problems presented in unstructured form.	30%	30%	30%

Examinations

- For International GCSE they are described in terms of subject matter

AO1 Demonstrate knowledge, understanding and skills in number and algebra:

- numbers and the numbering system
- calculations
- solving numerical problems
- equations, formulae and identities
- sequences, functions and graphs.

International GCSE has also Problem Solving and Mathematical Reasoning as overarching concepts

Examinations

- For International GCSE they are described in terms of subject matter

Mathematical Reasoning is not just about providing reasons (obviously!) and giving arguments:

• Showing working out (as in Simultaneous Equations)

• Drawing graphs

• Interpreting graphs (this could be a pictograph, for example)

Examinations

Demand is determined by:

- the complexity of elements of knowledge or task- linked to the expectation of the content standards of the qualification level
- the number of steps/linkages involved in a response
- the level of familiarity/prior knowledge students may have on the content or procedures required; the procedure is routine and does not require any adaptation or application
- the predictability of a question-from series to series - how familiar the question is over time
- the manner in which marks are awarded
- the use of verbs or command words - this is clearly a factor - but very dependent on the previous five above

Source: Pearson Edexcel Research Department

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Examinations

- Mathematics is about solving problems.
  Some of these ‘problems’ are relatively straightforward to solve, whilst others, either because of **unfamiliarity** or **complexity**, are very difficult.
- The following characteristics appear in problems (although not all in the same problem!)

A. There is little or no scaffolding: little guidance given to the student beyond a start point and a finish point. Questions do not explicitly state the mathematical process(es) required for the solution.

B. There is a need for multiple representations, such as the use of a sketch or a diagram as well as calculations.

C. The information is not given in mathematical form or in mathematical language; or there is a need for the results to be interpreted or methods evaluated, for example, in a real-world context.

D. There is a variety of techniques that could be used.

E. The solution requires understanding of the processes involved rather than just application of the techniques.

From the UK working party on A level mathematics, but also relevant to Int GCSE

Examinations

Activity 1

- Look at the questions on the sheet.
- Rank the questions in order of demand
- One set are A level
- One set are International GCSE

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Examinations

What happens in practice:

- All main content areas are tested nearly every paper
  All content areas are tested over a short period of time

- Each paper has AOs allocated roughly the same number of marks as previous papers of the same type

- Papers are set to cover the full demand range with lower demand generally at the start of the paper but also in the first parts of more complex questions nearer the end of the A level paper.

- For International GCSE there is an incline of difficulty from low to high through the paper.

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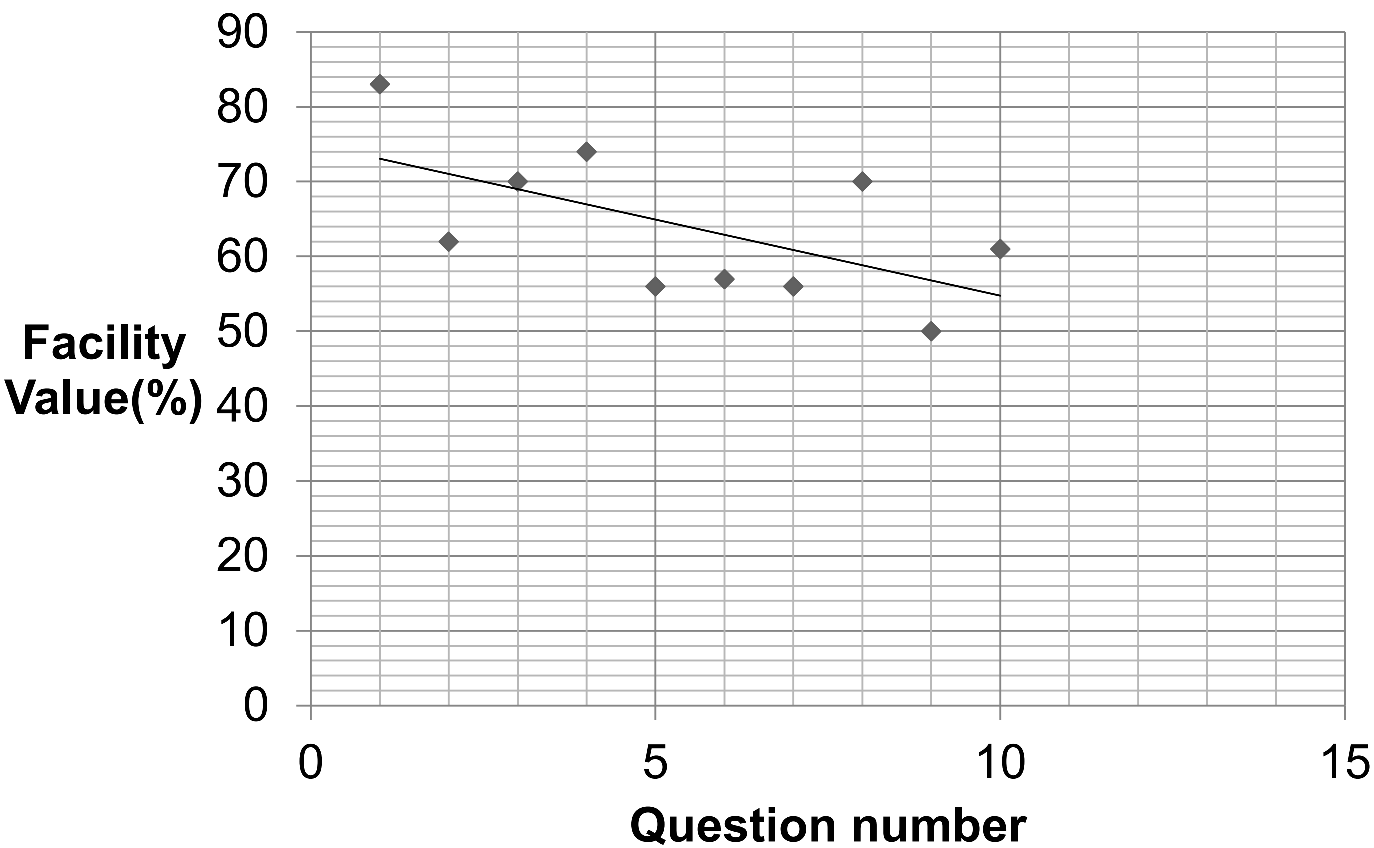
Examinations

- This shows the facility value of questions from June 2019 paper Pure1



- Note that questions tend to be ordered by number of marks

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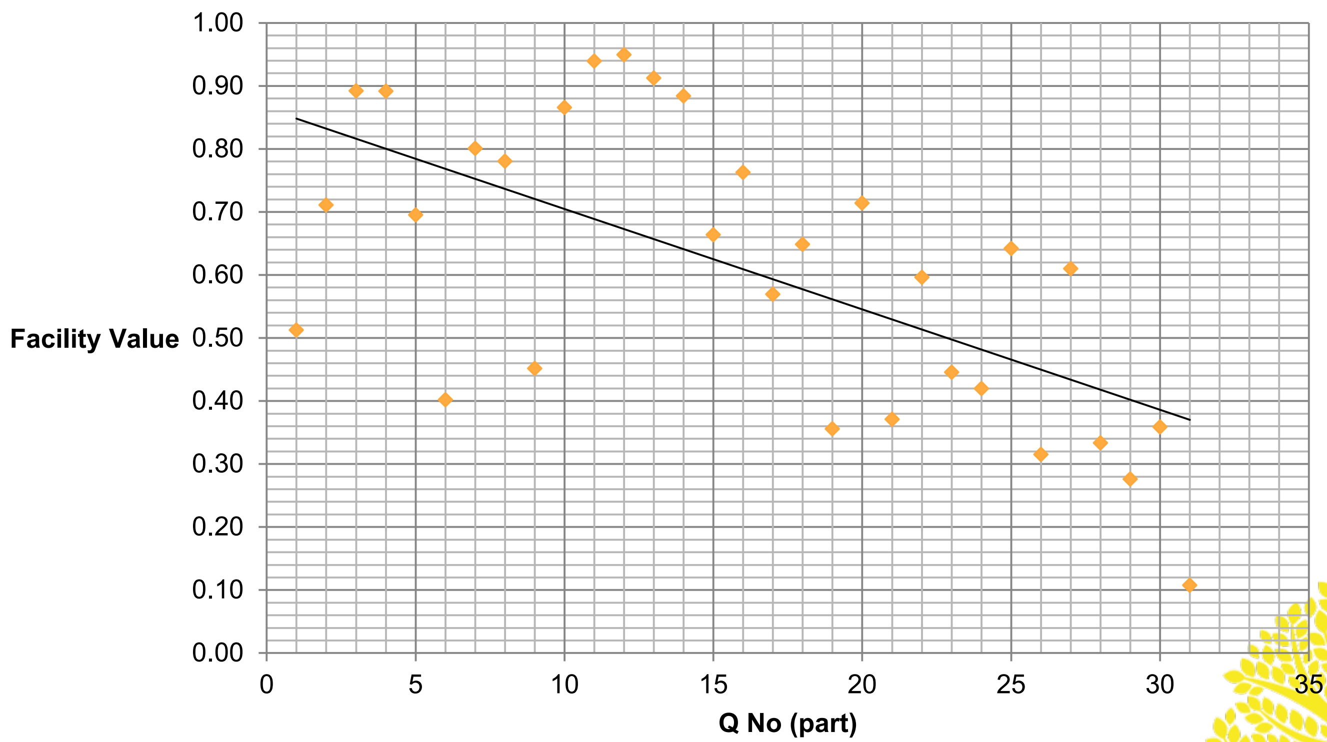
Examinations

- This shows the facility value of questions from June 2019 International GCSE paper



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Understanding mark schemes

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Examinations

- How mark schemes relate to the mathematical task.

The line with equation $y = 4x + c$, where c is a constant, meets the curve with equation $y = x(x - 3)$ at only one point.

(*a*) Find the value of c.

(4)

(*b*) Hence find the coordinates of the point of intersection.

(3)

(Total for Question 6 is 7 marks)

Think about the processes involved in doing this question.

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Examinations

- $x(x - 3) = 4x + c$
- $x^2 - 3x = 4x + c$
- $x^2 - 7x - c = 0$
- $\Delta = (-7)^2 + 4c = 0$
- $c = -49/4$

5 processes but only 4 marks

- $(x - 7/2)^2 = 0$
- $x = 7/2$
- $y = 7/2\,(7/2 - 3) = 7/4$

3 processes here?

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Examinations

- $x(x - 3) = 4x + c$
- $x^2 - 3x = 4x + c$
- $x^2 - 7x - c = 0$ M1
- $\Delta = (-7)^2 + 4c = 0$ dM1, A1
- $c = -49/4$ A1
- ………………………………………….
- $(x - 7/2)^2 = 0$ M1
- $x = 7/2$
- $y = 7/2\,(7/2 - 3) = 7/4$ M1 A1 for both values

Examinations mark schemes

- There are various types of marks and processes used to mark Edexcel IAL
- M marks – for correct methods (as defined by the mark scheme)
- A marks – for accurate and correct answers following a correct method

So M0A1 can NEVER be awarded

Also to get a method mark a process must be carried out
(Describing what you would do does **NOT** get the mark)

Examinations

Sometimes at the start of an answer

The others are

B marks – unconditional accuracy marks

dM marks – method marks that depend on the award of a previous method mark

In addition “ “ are used around values to denote answers which may be wrong but are carried through a solution. (‘follow though’ or ‘their’)

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Examinations

One concept and one process for the first mark

6.(a)	Sets $4x + c = x(x-3)$ and attempts to write as a 3TQ	M1
	Uses $b^2 = 4ac$ for their $x^2 - 7x - c = 0$	dM1
	Correct equation $49 = -4c$ or $49 + 4c = 0$	A1
	$c = -12.25$ oe	A1
		(4)

The first M must be earned.

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Examinations

(b)	Attempt to solve $x^2 - 7x - c = 0$ with their c Attempt to find the y coordinate for their x coordinate $\left(\frac{7}{2}, \frac{7}{4}\right)$ oe	M1 dM1 A1 (3)

‘Attempt’ - this is defined in the initial notes

The first M must be earned.

Examinations

How many marks is this worth?

Mark scheme and response is in your delegate book

$x(x-3) = 4x + c$

$x^2 - 3x = 4x + c$

$x^2 - 7x - c = 0$

$(-7)^2 - 4c = 0$

$c = 49/4$

....

$x^2 - 7x - 49/4 = 0$

$x = 8.45$

$y = 8.45(8.45 - 3) = 46.1$

?

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Examinations

International GCSE mark schemes are more straightforward with M, A and B marks along with the use of inverted commas to denote follow through.

Question	Working	Answer	Mark	Notes
10	$2 \times 3.50 + 4 \times 4.25$ (=24)	216	4	M1
	"24" − 7.60 (=16.4) or $\frac{\text{"24"}}{7.60} \times 100$ (=315.7..)			M1
	$\frac{\text{"16.4"}}{7.6} \times 100$ or "315.7" − 100			M1 for a complete method
				A1 for 215.7 – 216

It's important to know that the follow through on " 24" depends on the method in the line above being correct and not just on any old number

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Examinations

Activity 2

- Use the mark scheme provided to mark these samples of students’ work
- There are two questions from a recent Pure A level paper and one from a recent International GCSE paper

How students perform in exams

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Examination performance

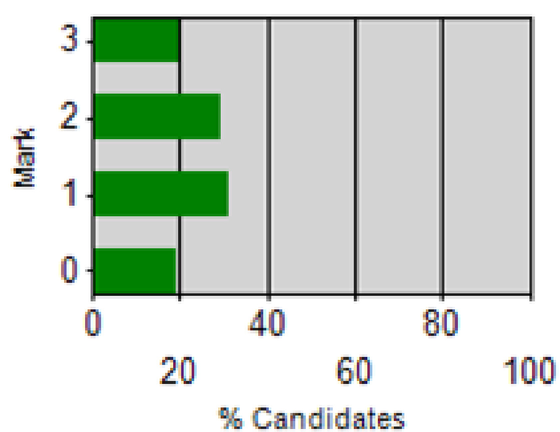
- Students have their individual strengths and weaknesses:
- Teachers have a good idea what grade E performances look like….
- …..and Grade A at A level

Examination performance

Q03

Mark	No Candidates	% Candidates
0	42	19.4%
1	67	31.0%
2	64	29.6%
3	43	19.9%
Total	216	

Grade E Above - 33 to 37 - Item: Q03



- Many students at this level made a good attempt.. .but nobody scored full marks
- Mean = 1.5

June 2019 Pure paper 2

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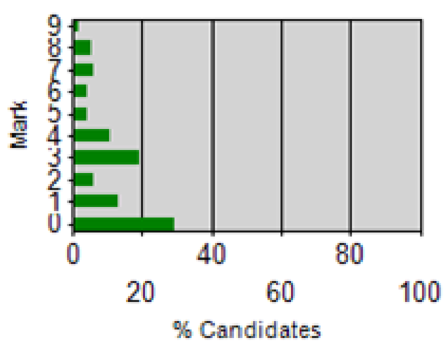
Examination performance

Q07

Mark	No Candidates	% Candidates
0	63	29.2%
1	29	13.4%
2	13	6.0%
3	42	19.4%
4	23	10.6%
5	9	4.2%
6	9	4.2%
7	13	6.0%
8	11	5.1%
9	4	1.9%
Total	216	

Grade E Above - 33 to 37 - Item: Q07



But found it harder here..........

Mean = 2.72

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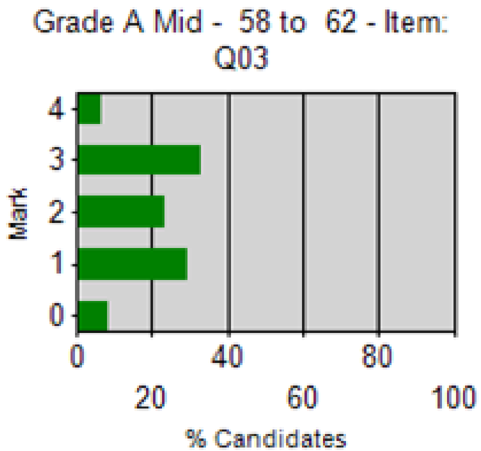
Examination performance

Q03

Mark	No Candidates	% Candidates
0	46	8.2%
1	165	29.5%
2	130	23.2%
3	184	32.9%
4	35	6.3%
Total	560	



- Grade A students did this…
- Mean = 1.99

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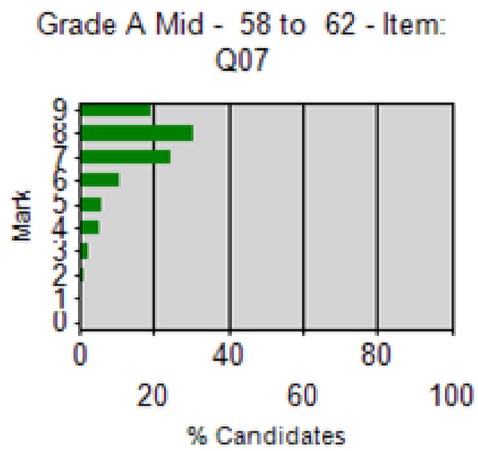
Examination performance

Q07

Mark	No Candidates	% Candidates
0	1	0.2%
1	4	0.7%
2	6	1.1%
3	12	2.1%
4	31	5.5%
5	32	5.7%
6	59	10.5%
7	138	24.6%
8	171	30.5%
9	106	18.9%
Total	560	

….. and did this

Mean = 7.10



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Examination performance

Grading

- Uses statistical evidence
- Uses professional expertise
- Each paper is graded separately
- Grades are tied to UMS and then marks can be read off the UMS scale

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Examination performance

The table below gives the lowest raw marks for the award of the stated uniform marks (UMS).

Module	80	70	60	50	40
WMA11 Pure Mathematics 1	58	51	44	38	32
WMA12 Pure Mathematics 2	60	53	46	40	34

	UMS	PM1	PM2	D1
	100	72–75	74–75	71–75
	84	61	63	
	83	60	62	61
	82			60
	81	59	61	
A	**80**	**58**	**60**	**59**
	79	57	59	
	78			58
	77	56	58	57
	76	55	57	
	75			56
	74	54	56	
	73	53	55	55
	72			54
	71	52	54	
B	**70**	**51**	**53**	**53**

Grading

- UMS is used to allow combinations of units (including resits) taken at different sessions

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Examination performance

The percentage of candidates obtaining at least the given number of uniform marks (UMS) at the time of grading are given below (the final figures may vary slightly from these).

Module	80	70	60	50	40
WMA11 Pure Mathematics 1	36.7	50.5	63.3	71.8	79.4
WMA12 Pure Mathematics 2	35.5	52.5	65.0	72.9	79.8
WDM11 Decision Mathematics 1	29.1	43.4	56.9	66.4	72.2

Grading

- Results from last summer for the new units……

Module	80	70	60	50	40
WFM01 Further Pure F1	53.6	68.7	79.4	86.5	90.5
WFM02 Further Pure F2	24.6	44.4	61.2	73.0	81.8
WFM03 Further Pure F3	43.2	60.2	70.2	77.2	83.8
WME01 Mechanics M1	22.1	35.4	49.0	59.1	69.1
WME02 Mechanics M2	44.6	56.9	67.6	76.5	83.1
WME03 Mechanics M3	26.9	46.5	59.9	70.1	77.7
WST01 Statistics S1	26.8	41.0	57.4	71.5	82.5
WST02 Statistics S2	39.7	57.9	70.3	79.5	86.3
WST03 Statistics S3	24.1	55.1	74.7	84.0	88.3
WDM01 Decision Maths D1	31.4	44.0	57.8	69.9	76.7

Grading

- …and the established ones

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Examination performance

Edexcel International GCSE Mathematics (4MA1)
Grade Boundaries – June 2019

Foundation Tier	9	8	7	6	5	4	3	2	1
Paper 1					70	59	43	27	12
Paper 2					64	55	41	27	12
Total					**134**	**114**	**84**	**54**	**24**

Higher Tier	9	8	7	6	5	4	3	2	1
Paper 1	76	64	52	41	31	21	15		
Paper 2	74	61	49	39	29	20	16		
Total	**150**	**125**	**101**	**81**	**61**	**41**	**31**		

(Total boundaries are given out of 200)

No UMS here – marks for the two papers are just added together

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Examination performance

Edexcel International GCSE Mathematics (4MA1(R))
Grade Boundaries – June 2019

Foundation Tier	9	8	7	6	5	4	3	2	1
Paper 1					71	58	42	27	12
Paper 2					69	56	42	27	12
Total					**140**	**114**	**84**	**54**	**24**

Date	9	8	7	6	5	4	3	2	1
Paper 1	79	64	49	38	28	18	13		
Paper 2	80	65	51	41	30	19	13		
Total	**159**	**129**	**100**	**79**	**58**	**37**	**26**		

(‘R’ papers are taken by those candidates in regions five or more hours ahead of GMT. Total boundaries are given out of 200)

The 4MA1(R) papers are graded to ensure comparability with 4MA1

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Examination performance

Mistakes students make that lose marks at A level

4. Find

$$\int \frac{4x^2 + 1}{2\sqrt{x}}\,\mathrm{d}x$$

giving the answer in its simplest form.

(5)

Just take a minute to work through this question taken from paper 1 June 2019

What mistakes did students make?

Examination performance

<u>Question 4 (Mean Mark 3.7 out of 5)</u>

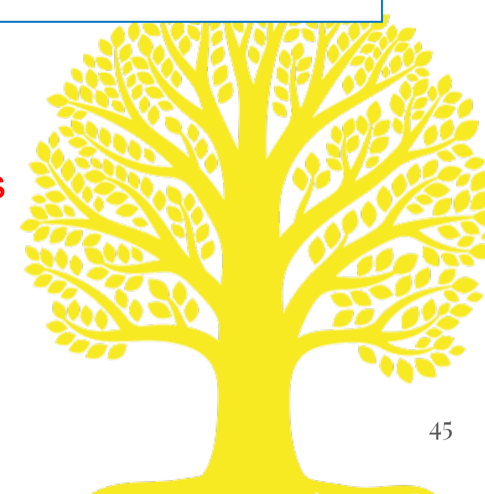
It was pleasing to note that most candidates knew that the given expression had to be written as a sum of terms before the integration was attempted. There is still a misconception, however, about how this should be carried out with many getting one of terms wrong.

Common errors included:

- $\dfrac{4x^2+1}{2\sqrt{x}} = 4x^2 + \dfrac{1}{2}x^{-\frac{1}{2}}$
- $\dfrac{4x^2+1}{2\sqrt{x}} = \left(4x^2+1\right)2x^{-\frac{1}{2}} = 8x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$

Once the correct sum was formed candidates generally performed the integration correctly with only a few making fractional or sign errors. The failure to add the constant of integration $+\ c$ was also seen.

Such specific comments may be especially useful for colleagues who are new to teaching



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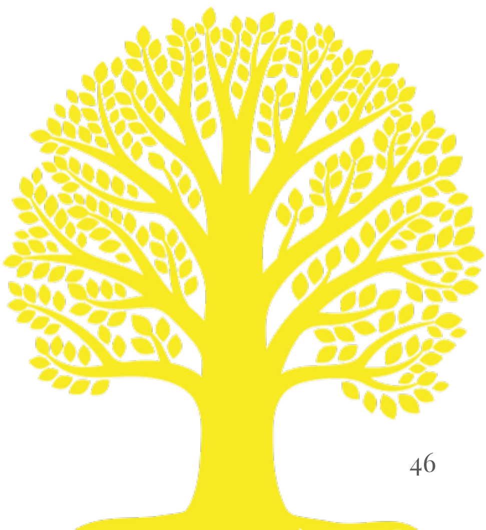
Examination performance

Activity 3

- Make a list of common errors that students often make at A level…..
- …..and/or for International GCSE
- There are lots, but try and get a minimum of five.



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Examination performance

Some common misconceptions which often appear in examiner reports (at A level)

Some of these are International GCSE too

Examination performance

…..and possibly how to avoid them….:

Build awareness of these into planning lessons:

e.g.. Write reciprocals as powers e.g.. 1/4x $= x^{-1}/4$*

Relate graphs of $y = x^2$ to graphs of $y = ax^2 + bx + c$*
Link repeated roots with tangency and show it

Ensure the use of the quadratic formula <u>correctly</u> – a common error is that the −b drifts away from the square root*
Also that their calculator probably works out -3^2 as −9*

Simultaneous equations, one quadratic – look at relationship with the graphs.
How does the number of solutions vary?
Successful students substitute and check in the original equations.*
When solving quadratic equations be clear about the range – $2x^2 - 17x + 36 < 0 \Rightarrow \quad x < 4.5, x < 4$

Check mode of calculator at the start of every test*

Make links $2x^2 - 17x + 36 = 0$ and $2y^4 - 17y^2 + 36 = 0$ *

Desmos is a free graphics program that students can access https://www.desmos.com/calculator

Similar points can be made for each unit (this one is Pure 1) by using examiner reports and the Edexcel SOW

The * denotes happens at INT GCSE too

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Examination performance

Some common misconceptions which often appear in examiner reports (at International GCSE)

Some of these are at A-Level too

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Examination performance

Some general advice from the A level papers:

Use of a formula

- Where a method involves using a formula that has been learnt, the advice is that the formula should be quoted first.

Normal marking procedure is as follows:

- **Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.**
- Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Examination performance

Some general advice:

Exact answers

- Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

- The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required

An example is shown on the next slide

Examination performance

2. Answer this question showing each stage of your working.

(a) Simplify $\dfrac{1}{4-2\sqrt{2}}$

giving your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers.

(2)

2.(a)	$\dfrac{1}{4-2\sqrt{2}} = \dfrac{1}{4-2\sqrt{2}} \times \dfrac{4+2\sqrt{2}}{4+2\sqrt{2}}$	M1
	$= \dfrac{4+2\sqrt{2}}{16-8} = \dfrac{1}{2} + \dfrac{1}{4}\sqrt{2}$ oe	A1

This step, or equivalent MUST be shown

This came from Pure 1, but it could be on an International GCSE paper

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Examination performance

Activity 4

- Make a list of common techniques or topics where students can use a modern (non symbol manipulating) calculator to reach an answer

- There are several, but try and get a minimum of four for **each** qualification (International GCSE and A-Level)

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Examination performance

Activity 4 – some ideas at A level

?

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Examination performance

Activity 4 – some ideas at International GCSE level

?

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Examination performance

Edexcel prevents too much reliance on calculators by:

- at A level – using an explicit instruction of what is not acceptable
- at International GCSE level – giving a direct instruction to **show working** clearly

'Show working clearly' does **not** mean that trial and error methods with answers shown, are acceptable.

Examination performance

Cognitive features that students need include:

- Good **long term** memory of facts and techniques
- An efficient **working** memory – interactions between facts and processes which use information from long term memory
- Problem-solving ability

Working memory is analogous to the CPU in a computer. Typically it can hold 5 to 8 'pieces' of data (e.g.. numbers)

Examination performance

Affective features that students need include:

Confidence – the belief that what you are doing is correct
the self-belief that you can do problems that look initially challenging

Persistence – the ability to keep working at a problem

Flexibility – knowing when to stop or backtrack on a problem

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Examination performance

9. A curve has equation

$$x = \frac{\sin y - \cos y}{\cos y + \sin y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(a) Show that the equation of the curve can be written as

$$x = \frac{-\cos 2y}{1 + \sin 2y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(b) Hence, or otherwise, show that

$$\frac{dx}{dy} = \frac{2}{1 + \sin 2y}$$

What knowledge is needed for a successful attempt at part (a)?

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Examination performance

Long term memory

It's obvious to us that the first step is to multiply numerator and denominator by $(\sin y + \cos y)$

Why do that?

Reason one

?

Reason two

?

Examination performance

9. A curve has equation

$$x = \frac{\sin y - \cos y}{\cos y + \sin y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(a) Show that the equation of the curve can be written as

$$x = \frac{-\cos 2y}{1 + \sin 2y} \qquad -\frac{\pi}{4} < y < \frac{3\pi}{4}$$

(b) Hence, or otherwise, show that

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2}{1 + \sin 2y}$$

Which is easier – the ‘hence’ or the ‘otherwise’?

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Examination performance

Hence:

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{(1 + \sin 2\mathrm{y})(- - 2\sin 2\mathrm{y}) - 2\cos 2\mathrm{y}(-\cos 2\mathrm{y})}{(1 + \sin 2\mathrm{y})^2}$$

What long term memory must students have here?

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2(\sin 2\mathrm{y} + \sin^2 2\mathrm{y} + \cos^2 2\mathrm{y})}{(1 + \sin 2\mathrm{y})^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2(\sin 2\mathrm{y} + 1)}{(1 + \sin 2\mathrm{y})^2}$$

etc

Pearson Edexcel

Examination performance

Otherwise:

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{(\text{cosy} + \text{siny})(\text{cosy} - -\text{siny}) - (\text{siny} - \text{cosy})(-\text{siny} + \text{cosy})}{(\text{cosy} + \text{siny})^2}$$

What long term memory must students have here?

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{(\text{cosy} + \text{siny})(\text{cosy} + \text{siny}) + (\text{siny} - \text{cosy})(\text{siny} - \text{cosy})}{(\text{cosy} + \text{siny})^2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{2\text{sin}^2\text{y} + 2\text{cos}^2\text{y}}{\text{sin}^2\text{y} + \text{cos}^2\text{y} + 2\text{sinycosy}}$$

etc

Examination performance

Activity 5

Look at Practice paper Pure 3 and International GCSE paper 1H

What are the long term memory requirements of the questions?
(Concentrate on questions 4, 5 and 6 from Pure 3
and 13,15 and 16 from 1H)

What can be obtained from the Formula booklet (formula page)?
Should we insist that students learn formula even if they are in the formula book (or in the case of INT GCSE, on the formula page)?

Discuss with other delegates

Examination performance

Improving long term memory in mathematics:

- Improving knowledge of formulae
- Improving knowledge of procedures

Examination performance

Improving long term memory in mathematics:

- Improving knowledge of formulas

- Enforce learning of formulas even if in the formula book (or page)

e.g. the expansion of sin (A + B)

- Expect students to learn alternative versions of standard formulae (that may be in the formula book/page)

e.g. The alternative version of the cosine rule i.e. the cos *A* = form

Examination performance

Improving long term memory in mathematics:

- Improving knowledge of formulas

Do this by having short tests very often (you may have to convince some students that learning extra material, some of which is in the formula book/page, will lead to better results)

You could build in such tests into your SOW and/or lesson plans

Pearson Edexcel

Examination performance

Improving long term memory in mathematics:

- Improve knowledge of standard procedures

Again short tests can be useful

Items such as $4^{-1.5}$ = ? (without a calculator)

$(x + 4)^2 – (x-3)^2 =$

A hemisphere has a radius of 30 cm. Find its volume

Key idea is FLUENCY

Examination performance

Improving long term memory in mathematics:

- Improve knowledge of standard procedures

Again, short tests can be useful

Items such as: Solve $x(x – 4) = 2x^2$

Solve $\sin (2x + 40)^o = 0.5$ (i.e. where mistakes are common)

Find an expression equal to $(\sin x + \cos x)^2$

i.e. Where students have yet to make connections between trig functions

Key idea is FLUENCY

Examination performance

Working memory in mathematics:

Increasing evidence that:

- Working memory plays a crucial role in mathematics (for all ages)
- Working memory can be improved through training (for all ages)

Why is it important:

'Problems often involve multiple steps with intermediate results that need to be kept in mind temporarily to accomplish the task at hand successfully.'

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Examination performance

Improving working memory in mathematics:

- Focus on the way in which students can use the facts they have learned to deal with new (possibly very unfamiliar) problems.

- Ask students to ‘think aloud’ when doing a question in class.

- Ask students to do a question without (or with minimal) writing down.

For example at GCSE:
‘There are 6 green and 4 red counters in a box.
Two counters are taken at random.
What is the probability they are different colours?

Examination performance

Improving working memory in mathematics:

- There are many websites which claim to improve working memory
- Some are free but many require a subscription/payment
- Some have little research to back up their claims to improve brain function

Examination performance

Improving short term memory in mathematics:

Pearson has its own suite of online activities which do help to improve brain function:

- Originally developed to help those with cognitive impairment (ADHD, dementia) as a support for medical/psychological interventions
- Subsequently extended to support all children and adults.

https://www.pearsonclinical.co.uk/Cogmed/Cogmed-Working-Memory-Training.aspx

Pearson Edexcel

Examination performance

- Support / FAQs
- Resources & Reviews
- Video Presentations

Improving short term memory in mathematics:

Cogmed is an online resource; its value is robustly supported by many studies

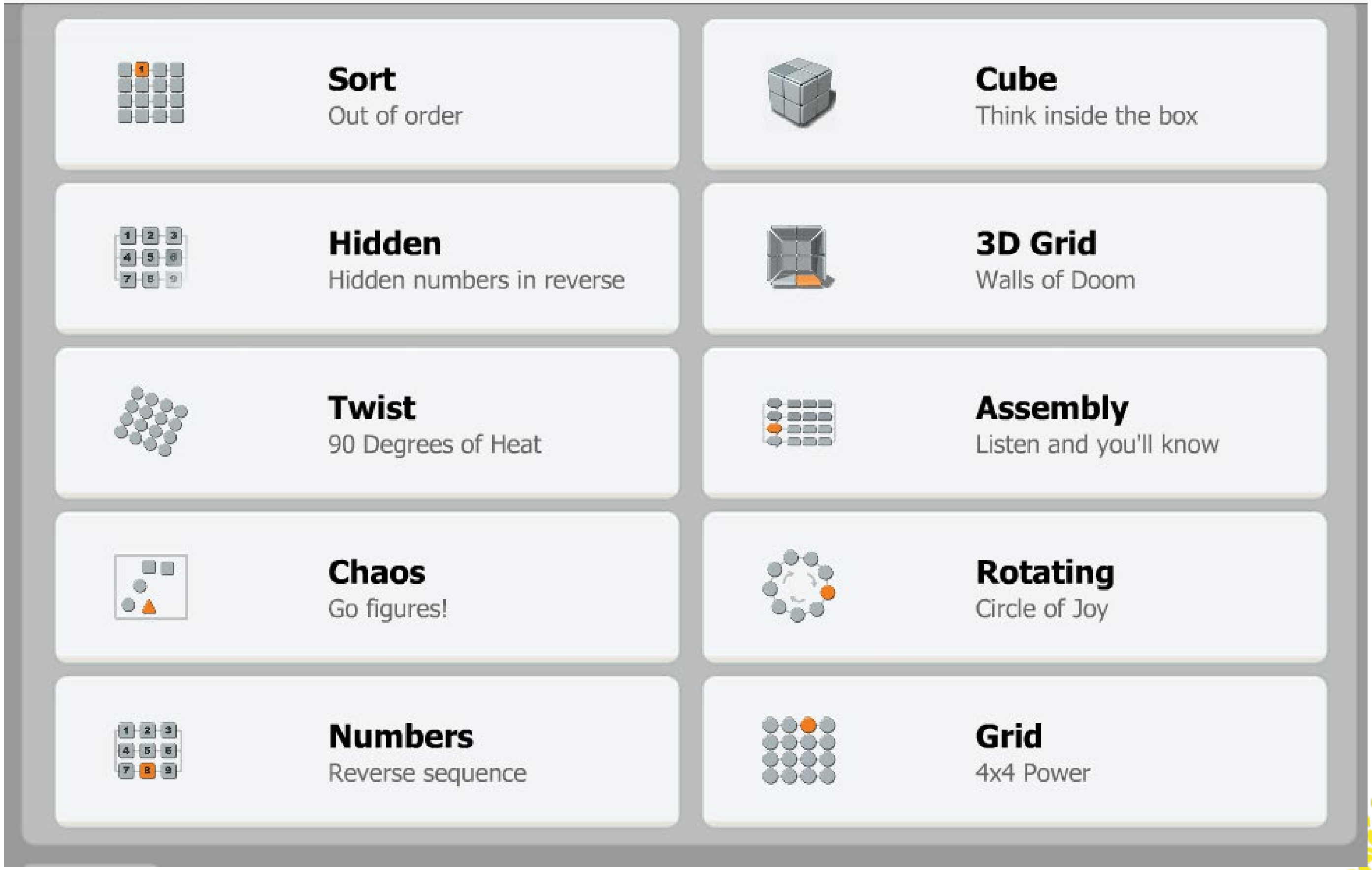
What is Cogmed Working Memory Training?

Cogmed Working Memory Training is a computer-based solution for improving attention by increasing working memory capacity over a five week training period.

Through combining cognitive neuroscience with innovative computer game design and close professional support, Cogmed can deliver substantial and lasting benefits to its users:

- Aims to improve attention by increasing working memory capacity
- An online, coach-supported training program
- Can be completed in around five weeks in school, home or work environments
- For children and adults alike
- Based on solid research.

Examination performance

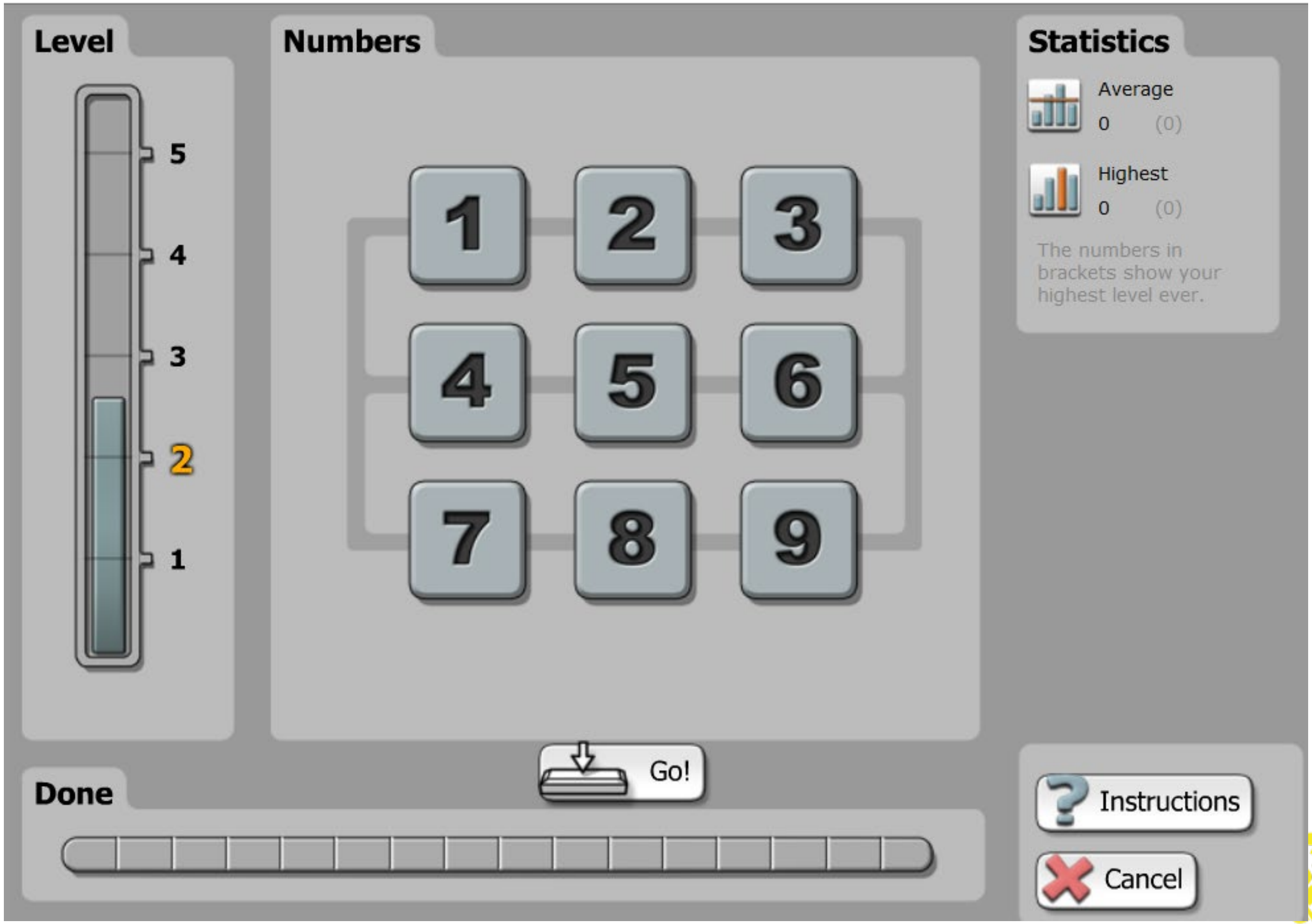


Improving short term memory in mathematics:

Cogmed front page

Pearson Edexcel

Examination performance



In this task, up to 5 numbers are lit up in sequence

Then you have to key in the 5 numbers in reverse

Pearson Edexcel

Examination performance

Centres have to pay for Cogmed.

A good working memory is necessary for most subjects………

……so the cost could come out of the school budget, rather than the mathematics department one!

Pearson Edexcel

Examination performance

A recap of what is a problem.

A. Little or no scaffolding; questions do not explicitly state the mathematical process(es) required for the solution.

B. Requires multiple representations, such as the use of a sketch or a diagram as well as calculations.

C. The information has to be translated into suitable mathematical language

D. There is a variety of techniques that could be used.

E. The solution requires understanding of the processes involved rather than just application of the techniques.

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Examination performance

Here is the first example. (A level)

Problem solving exemplar 1

A circle, centre C (1, 1), touches both axes, as shown in Fig. 1. AB is a tangent to the circle. The triangle OAB is isosceles.

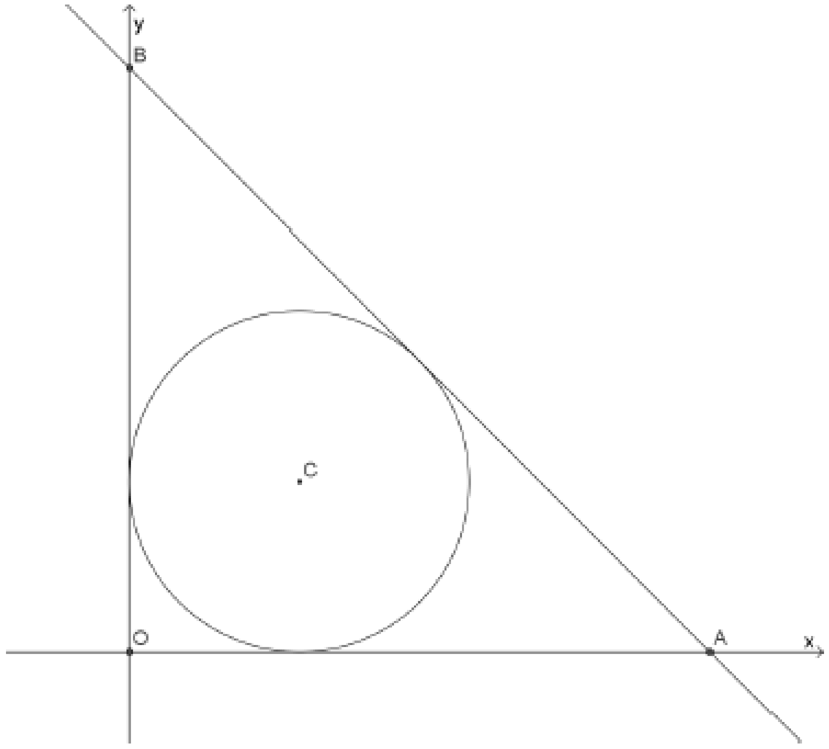

Fig. 1

Find the equation of AB, giving your answer in exact form. **[8]**

No structure (apart from the diagram) given
Style of demand probably requires surds.

Examination performance

Here is the first example.

Problem solving exemplar 1

A circle, centre C (1, 1), touches both axes, as shown in Fig. 1. AB is a tangent to the circle. The triangle OAB is isosceles.

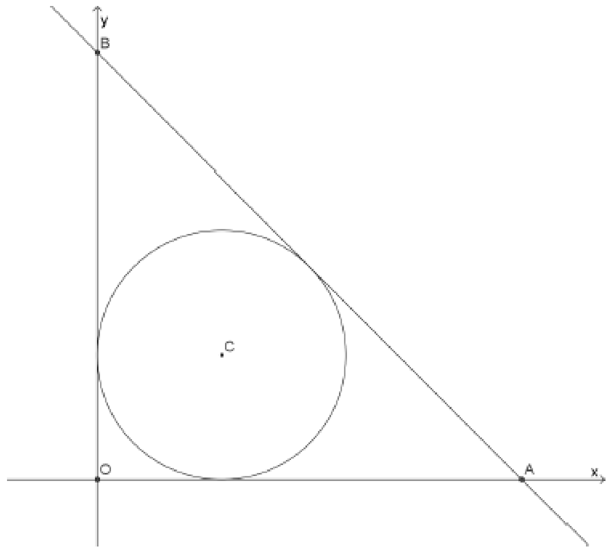

Fig. 1

Find the equation of AB, giving your answer in exact form. **[8]**

?

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Examination performance

Here is a second example. A level

A curve is defined by the parametric equations

$$x = 3 + 2t\,,\quad y = 2 - \frac{3}{t}$$

Find the value of t at the point where the normal to the curve at (9, 1) crosses the curve again. **[9 marks]**

Make a plan

?

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Examination performance

Here is a third example. International GCSE level

5 120 children go on an activity holiday.
The ratio of the number of girls to the number of boys is 3 : 5

On Sunday, all the children either go sailing or go climbing.

$\frac{16}{25}$ of the boys go climbing.

Twice as many girls go sailing as go climbing.

Work out how many children go sailing on Sunday.

Make a plan

?

Pearson Edexcel

Examination performance

Thank back to the GCSE example

Features for successful problem solving in examinations:

- Having an outline plan
- Knowing the subject matter really well (to make the plan work)
- Carrying out the plan - best to show working line by line
- Trying to anticipate whether the plan is going in the right direction
- Asking: is the answer reasonable and has all the information been used?

Examination performance

Features for successful problem solving in examinations

The more familiar the question is to the student the more likely they are to be able to make a good attempt – especially at getting started.

So they need plenty of practice!

So the first thing that students should do when meeting questions is ask:

Have I seen anything like it before?
If I have, what did I do to solve it?

Examination performance

Features for successful problem solving in examinations

The more familiar the question is to the student the more likely they are to be able to make a good attempt – especially at getting started.

So they need plenty of practice!

The functions f and g are such that

$$f(x) = x^2 - 2x \qquad g(x) = x + 3$$

The function h is such that $h(x) = fg(x)$ for $x \geqslant -2$

Express the inverse function $h^{-1}(x)$ in the form $h^{-1}(x) = \dots$

Q21 of Jan 20 was done much better than Q22 of June 19

The function g is such that $g(x) = 2x^2 - 20x + 9$ where $x \geqslant 5$

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \dots$

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Examination performance

Activity 6

There are 3 questions taken from a practice Pure paper 4

Write down a plan for the completion of each of the questions

Alternatively/ additionally write one for Q22 of International GCSE Jan 2020 Paper 1H (the paper you looked at for Activity 5) (it’s quite challenging!)

Examination performance

‘Proving’ and ‘Showing that’

Proof appears in Pure 2 and Pure 4

‘Show that’ can appear in any unit.

Proof requires a more formal structure than ‘ Show that’

At International GCSE it often appears in questions which could be worked out trivially using a scientific calculator.

It also appears at International GCSE where students are asked to give reasons in geometrical questions (typically those involving circle geometry) or to demonstrate the truth of an algebraic statement.

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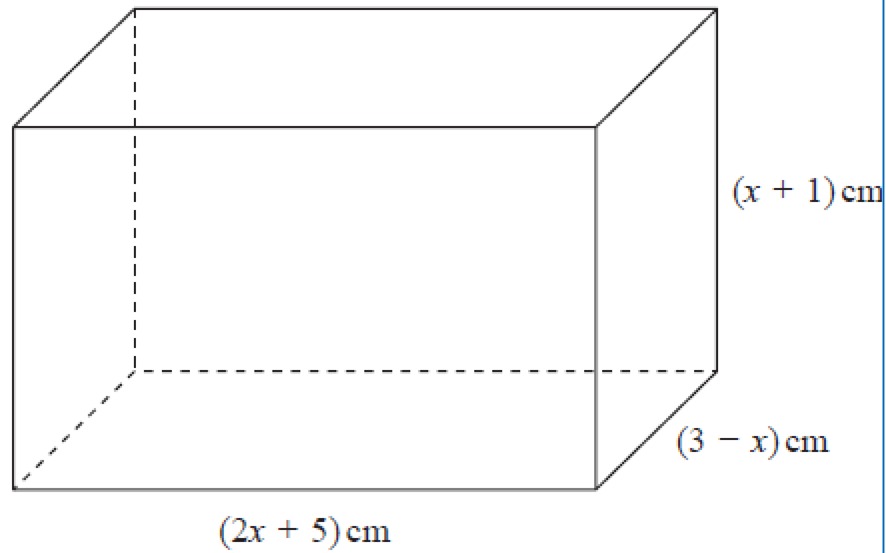
Examination performance

‘Show that’
Jan 2020 paper 1H

15



The diagram shows a cuboid of volume V cm^3

(a) Show that $V = 15 + 16x - x^2 - 2x^3$

Write down initially the volume of the cuboid – then expand the brackets and collect terms.

Examination performance

‘Proving’

- Write down that which has to be proved
- Translate, if necessary, into a mathematical form
- Carry out the mathematical manipulation
- Relate the outcome back to the initial statement

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Examination performance

'Proving'

Example:

Show that $e^x > 1 + x$ for $x > 0$

?

Examination performance

'Proving'

There are also specific techniques that students have to know.

- Disproof by <u>counterexample</u>
- Proof by <u>contradiction</u>

Examination performance

‘Proving’

There are also specific techniques that students have to know.

Disproof by <u>counterexample</u>

e.g. Give a counterexample to the conjecture that $10^n > n^{10}$ where n is a natural number

One counterexample is

?

So the conjecture is not true

‘Counterexample can also appear on International GCSE but **not** using that term

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Examination performance

‘Proving’

There are also specific techniques that students have to know.

Proof by <u>contradiction</u>

e.g. The Fibonacci sequence begins 1 1 2 3 5 8 with the sum of the last two terms giving the next term.

Prove, by contradiction that there are an infinite number of terms which are divisible by 3

How do we start?

Examination performance

‘Proving’

Assume the contrary. There are a finite number of terms divisible by 3.

Let the largest term divisible by 3 be 3r

The next term must be 3k + 1 or 3k + 2

Case 1 : the next term is 3k +1 .

?

Case 2 : the next term is 3k +2 .

Works similarly

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Examination performance

‘Proving’

Proof by contradiction

• Start by stating an assumption which is the logical negative of the statement to be proved

• Translate, if necessary, into algebraic form

• Carry out algebraic manipulation….

• ….to reach a conclusion which is logically impossible or contradicts the logical negative

• STATE this and conclude that the original statement is true

Examination performance

'Proving'

If you can't be elegant, do something!

e.g. Show that $f(n) = n\,(n+1)(n+2) - (n-1)n(n+1) = 3n\,(n+1)$

Method 1

?

Method 2

?

Organising teaching to raise achievement

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Teaching and Learning

Features that have an effect on organisation include:

- Number of hours contact time per course (per week)
- Class size
- Recruitment of students policy on to A level
- Whether classes are shared or not
- Student motivation

Teaching and Learning

Features include:

- Planning the course

Activity 7 – This asks various questions on the factors within which you have to work, such as contact time.

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Teaching and Learning

Student motivation

Across most countries, a strong disciplinary climate is consistently and robustly associated with better performance.*

N.B. Yr 12 A level students are just a little older than Yr 11 students!

- As stated earlier, being clear about expectations in terms of punctuality, work completion and exam/test preparation is very important.
- Share objectives and the timescale with the students so they know what (approximately) is going to be done when (approximately).
- Give feedback often and make it supportive – suggest where and how the student can improve.

* Pisa available from https://www.oecd-ilibrary.org/docserver/9789264039520-en.pdf?expires=1576523109&id=id&accname=guest&checksum=8561ED821869803C03DC158D296E6EDB

Teaching and Learning

More subject specific points:

- Insist that students learn facts and formulae even if in the formula book – it will increase their fluency when it comes to solving problems
- Model the solution of standard questions; set similar questions and review. The aim is that all students understand the subject matter
- For a ‘new’ topic, show how it fits in with previous topics – review and revise if necessary
- Give tasks for which there is more than one approach. Share these with all members of the class.

Use ICT as a time saver and expect students to do the same.

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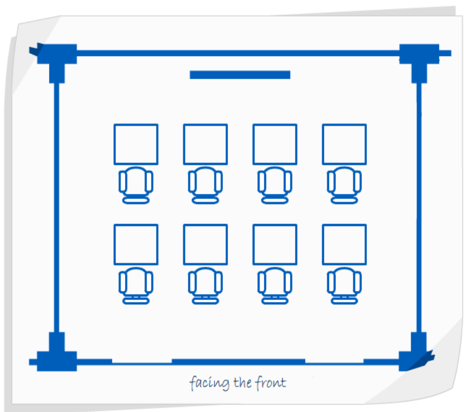
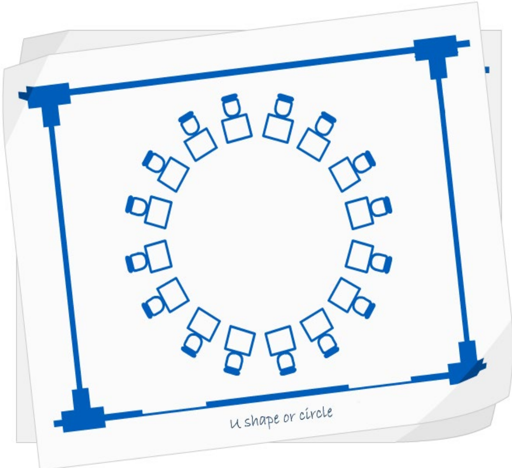
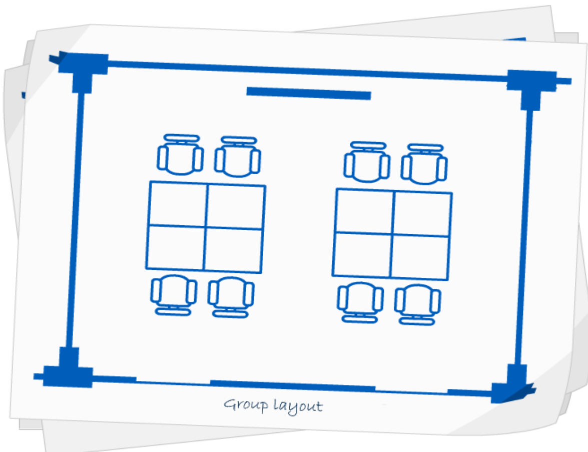
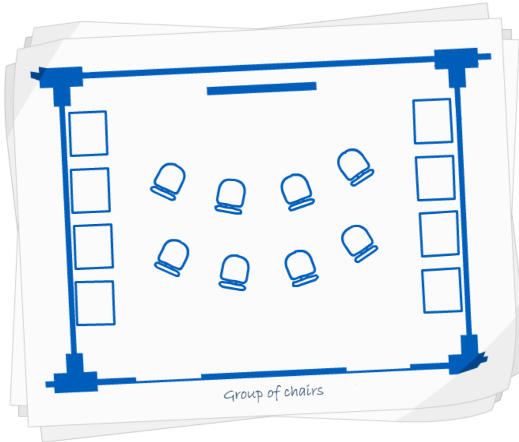
Teaching and Learning

Learning spaces









Teaching and Learning

Features include:

Activity 8 – Which classroom layout do you use?

What are the advantages and disadvantages of each layout?

Is there a school policy on classroom organisation?

No written work this time – discuss with other delegates

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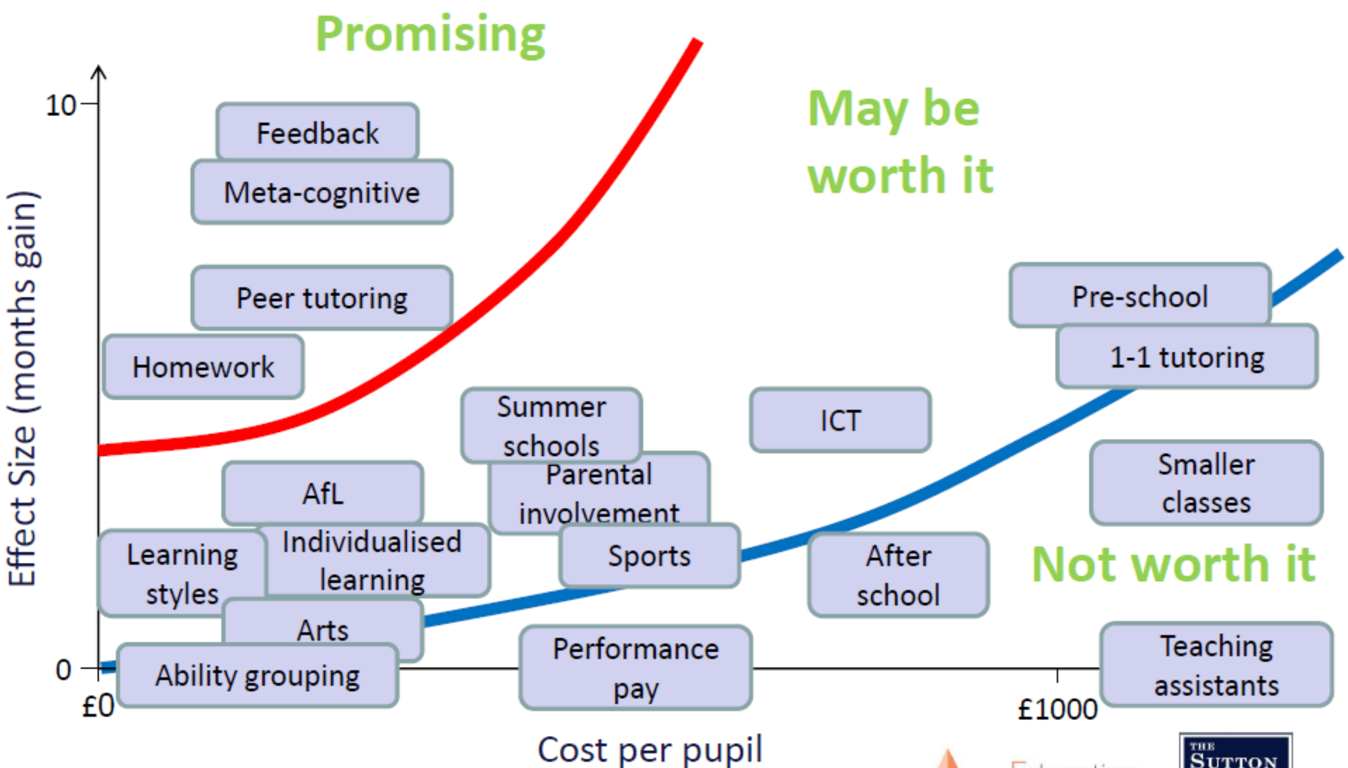
Teaching and Learning

Costs in 2013

Overview of value for money



Education Endowment Foundation

The Sutton Trust

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Teaching and Learning

What seems to work:

Metacognitive learning

- This should be a school-wide process
- It contributes to the development of transferable skills and independent learning in students

The need for transferable skills

In recent years, higher-education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.
The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as 'the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully perform…….

Metacognitive learning corresponds to the cognitive domain

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Teaching and Learning

What seems to work:

Metacognitive learning

- Students **plan** how to approach a given task
- Students **monitor** their comprehension of the task
- Students **evaluate** their progress towards the completion of the task
- Students **reflect** on how they completed the task.

These are similar to the skills we want students to have when solving problems in mathematics

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Teaching and Learning

What seems to work:

Feedback

- Written in terms of ‘how to improve’ when returning marked homework….
  ….. And then following up to see if they have

- Showing appreciation of ideas and answers in class even if they are wrong and trying to get a student to see why they are

- Test and mock exam marking

tell them **how** they got it wrong
tell/ask **why** they got it wrong
tell/ask **how** can they get it right

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Teaching and Learning

What seems to work:

Feedback from tests and exams

Source; CEM centre
Durham University (UK)

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Teaching and Learning

What seems to work:

Feedback from tests and exams

Ways in which valid and reliable questions can be obtained include:

- examWizard
- ResultsPlus
- both free to Edexcel centres

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Teaching and Learning

What seems to work:

Feedback from tests and exams

- examWizard
- free online tool
- bank of past exam questions organised by topic
- enables a teacher to build own test paper on a particular topic
- can be exported as a Word document

https://www.examwizard.co.uk/

Teaching and Learning

What seems to work:

Feedback from exams

ResultsPlus:

- free online tool

- can be used to analyse mock results and compare them(anonymously) with those of other schools

- based on previous exams so can be compared with actual results

- mock papers are marked at the centre

https://www.youtube.com/watch?v=2bpOI5sHzjA

https://qualifications.pearson.com/en/support/Services/ResultsPlus.html

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Teaching and Learning

What seems to work:

Peer Tutoring

• either formally – one student instructs at least one other

• or informally – students listen in class to ideas/explanations of others

But also

• 1 to 1 or small group tutoring – can be cost free in a boarding school.

Teaching and Learning

Planning the course

- Work to key dates – have a departmental schedule
- Share key dates with students - ensure they have them in a diary and that key dates are on the school website
- Decide on the order of teaching the units – one teacher or two teachers

Teacher A Pure 1 followed by Pure 2
Teacher B Mechanics 1 or Statistics 1 or Decision Maths 1

Teacher A Pure 1
Teacher B Mechanics 1 etc
Teacher A and B share Pure 2

Teaching and Learning

Planning the course

Have a scheme of work

There are good Edexcel ones to use or consult

- Objectives
- Prior knowledge
- Teaching points
- Opportunities for Reasoning/Problem solving
- Common misconceptions

https://qualifications.pearson.com/content/dam/pdf/International%20Advanced%20Level/Mathematics/2018/Teaching-and-Learning-Materials/Scheme-of-Work-Pure-Maths-1.pdf

https://qualifications.pearson.com/en/qualifications/edexcel-international-gcses-and-edexcel-certificates/international-gcse-mathematics-a-2016.coursematerials.html#filterQuery=Pearson-UK:Category%2FTeaching-and-learning-materials

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Teaching and Learning

Scheme of Work – Overview – translate into weeks at school

Mechanics 1

Unit		Title	Estimated hours
1		**Quantities and units in mechanics:** Introduction to mathematical modelling and standard S.I. units of length, time and mass	1
2		**Vectors in mechanics**	
	a	Definitions, magnitude/direction, addition and scalar multiplication	7
	b	Position vectors, distance between two points, application of vectors to displacement, velocity, acceleration and forces	7
3		**Kinematics of a particle moving in a straight line**	
	a	Graphical representation of velocity, acceleration and displacement	5
	b	Motion in a straight line under constant acceleration; *suvat* formulae for constant acceleration; Vertical motion under gravity	6
4		**Forces and Newton's laws**	
	a	Newton's first law, Newton's third law, force diagrams	3
	b	Newton's second law, 'F = ma', resolving forces, connected particles, problems involving smooth pulleys	8
	c	Momentum and impulse; derivation of units and formulae Impulse-momentum principle. Conservation of momentum applied to collisions and 'jerking' string problems	8
	d	Friction forces (including coefficient of friction μ)	4
5		**Statics of a particle:** Equilibrium, Forces in vector form, Maximum value of the frictional force	4
6		**Moments:** Forces' turning effects	7
			60 hours

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Teaching and Learning

Excerpt from Mechanics 1

OBJECTIVES

By the end of the sub-unit, students should:

- recognise when it is appropriate to use the *suvat* formulae for constant acceleration;
- be able to solve kinematics problems using constant acceleration formulae;
- be able to solve problems involving vertical motion under gravity.

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Teaching and Learning

Excerpt from Mechanics 1

TEACHING POINTS

Make links back to Unit 3a and contrast the previous graphical approach with this algebraic approach. Note that there are five quantities, *s, u, v, a* and *t* (four vectors and one scalar) and each formula relates four of them hence there are five formulae. The formulae that must be derived and learnt are:

- $v = u + at$
- $s = \frac{(u+v)t}{2}$
- $s = ut + \frac{1}{2}at^2$
- $v^2 = u^2 + 2as$
- $s = vt - \frac{1}{2}at^2$

These formulae are only valid for *constant* acceleration in a straight line (and are referred to as the *suvat* formulae).

When solving problems, write down known variables and the variable(s) to be found – this should help to identify which one (or more, as some problems will involve simultaneous equations) of the *suvat* formulae to select. Emphasise to students the need to make sure units are compatible.

Model the good practice of drawing a diagram to illustrate the situation whenever possible, especially when considering vertical motion under gravity. This will encourage students to draw their own diagrams.

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Teaching and Learning

Excerpt from Mechanics 1

OPPORTUNITIES FOR PROBLEM SOLVING/MODELLING

One of the more demanding problems is when two objects are released (or dropped) at different times, say 2 seconds apart, and students are asked to find the common position when one catches-up or passes the other. Students may find it difficult to select the times (values of t) to assign in the equations; they may need guiding towards t and $(t-2)$ or t and $(t+2)$.

COMMON MISCONCEPTIONS/EXAMINER REPORT QUOTES

Students are generally able to use *suvat* formulae in 2D to find unknown heights, velocities etc. However, students sometimes ignore the significance of a negative value for velocity, acceleration or displacement and don't refer their answer back to the original problem. They need to recognise that $s = -3$ m means the object is 3 m *below* its starting point in the negative direction i.e. s is effectively a coordinate. This is where a diagram helps students understand the physics of the situation.

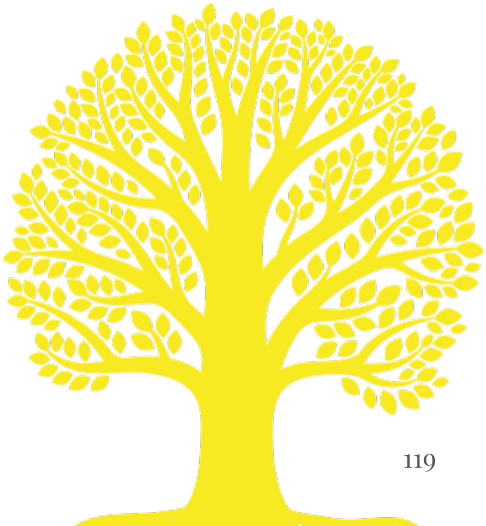
Teaching and Learning

For your most mathematically able students in sixth form

Consider:

- Edexcel's AEA paper - no additional content, but deeper problems (and marks for insight and awareness)

- Edexcel's Extended Project Qualification - research and inquiry carried out by student.



Teaching and Learning

For your most mathematically able students:

- Edexcel's AEA paper - 3 hours, 7 questions, total of 100 marks.

Questions totals ranging from about 5 to about 20.

Two grades – Merit and Distinction.

Mainly pure maths with a little mechanics and statistics.

No Further Maths knowledge required.

https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html

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Teaching and Learning

For your most mathematically able students:

Edexcel's Extended Project Qualification

The Extended Project is a standalone qualification that can be taken alongside International Advanced Level (IAL) qualifications. It supports the development of independent learning skills and helps to prepare students for their next step – whether that be higher education or employment. The qualification:

- is recognised by higher education for the skills it develops
- is worth half of an International Advanced Level (IAL) qualification at grades A*–E
- carries UCAS points for university entry.

The Extended Project encourages students to develop skills in the following areas: research, critical thinking, extended writing and project management. Students identify and agree a topic area of their choice for in-depth study (which may or may not be related to an IAL subject they are already studying), guided by their teacher.

https://qualifications.pearson.com/en/qualifications/edexcel-project-qualification/level-3.html

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Teaching and Learning

Using Pearson support to enhance student outcomes.

Many international centres already use ResultsPlus as a tool to analyse the performance of their students.

For a unitised course this can be used usefully as a teaching resource:

• Use Access to Scripts Service (ATS) to look at the actual answers of a student.

• (Decide whether to ask for a re-mark)

• Use ResultsPlus for one or more student(s) to compare their performance with the general entry.

• Use examWizard to produce targeted practice material for resit student(s)

All these features are free to Edexcel centres

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Edexcel support to help raise achievement

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Support Overview

Free Support

Getting Started Guide & Scheme of Work	Getting Ready to Teach Events	Subject interpretation of transferable skills
Subject Advisor	**Results Plus**	Regional Support Manager

Additional support for selected subjects

Curriculum Matched Publishing	Lesson plans	Exemplar Marked Responses
Topic booklets & Subject guides	Additional SAMs	**Exam Wizard**

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Other useful links

1. **Grade Boundaries**

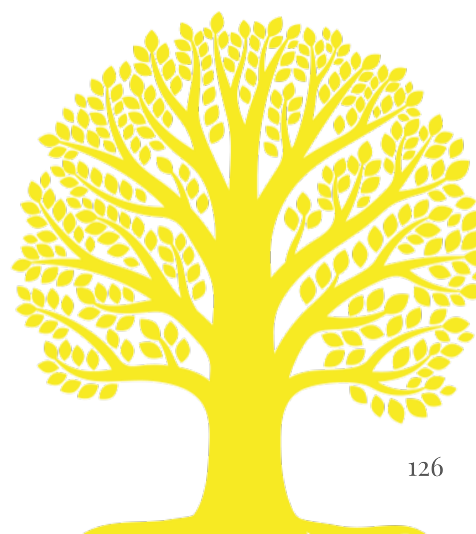
This page shows the minimum marks needed to achieve a certain grade for all UK and international examinations. Also refer to the examiners report which is available for download with other documents.

2. **Examination Results Statistics**

Results statistics summarise the overall grade outcomes of candidates sitting Pearson Edexcel examinations.

3. **Progress to University**

Here you can find information and guidance about how to progress to universities worldwide with Pearson Edexcel qualifications.



Other useful links

Maths Emporium

- Free to Edexcel centres https://www.mathsemporium.com/mathematics-emporium/?redirect_to=https%3A%2F%2Fwww.mathsemporium.com%2F
- All papers in all Edxcel mathematics examinations back to 2000
- Mark schemes
- Examiner reports
- Specimen and practice papers and mark schemes
- Grade boundaries and grade statistics

Contact your dedicated Subject Advisor

Subject Advisor details

Your subject advisor is **Graham Cumming**
Phone: **+ 44 (0)20 7010 2174**
Twitter: **@EmporiumMaths**
Email: Teachingmaths@pearson.com

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